LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2023**

UMT 4601 – COMBINATORICS

Date: 06-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

	SECTION A - K1 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
1.	**Answer the following**
a)	Define generating function.
b)	There are five seats in a row available, but 12 people to choose from. How many different seating's are possible?
c)	10 people meet and form 5 pairs. In how many ways their pairs can obtain?
d)	When a path or trial is said to be closed?
e)	Write the rook polynomial of an 2×2 board.
2.	**Fill in the blanks**
a)	If $a_n = a_{n-1} + n$. If $a_1 = 0$. Then $a_n =$____________.
b)	$(1+x)^3 =$ __________________________
c)	A beats B, A beats C, B beats C, D beats A, B beats D, D beats C, The score sequence of this game is ____________________
d)	A connected graph with no cycles is called _______________
e)	If a $n \times m$ board has the _____________________, then it is said to have a forbidden position
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**MCQ**
a)	$f(4,2) =$____________________. (a) 10 (b) 11 (c) 12 (d) 13
b)	__________ number of necklaces can be designed from n colours, using one bead of each colour. (a) $\frac{1}{2}n!$ (b) $\frac{1}{2}(n-1)!$ (c) $(n-1)!$ (d) $\frac{1}{6}(n-1)!$
c)	The derangement of 1 2 3 is ________ (a) 1 2 1 (b) 3 2 2 (c) 2 3 1 (d) 1 2 2
d)	The number of edges in a walk is called ____________________. (a) length of the walk (b) identical walk (c) non-identical walk (d) None
e)	$A = \{1,2,3\}$ and $B = \{2,3,4\}$; Then $\|A \cup B\| =$ ___________. (a) 1 (b) 2 (c) 3 (d) 4
4.	**True or False**
a)	An equation that defines recursively a sequence with one or more boundary conditions are said to be a recurrence relation.
b)	In 12 tone music, the 12 notes of the chromatic scale are put in a row, and then there are 12! Number of possible rows which have to be played in that particular order
c)	In a graph, root of the tree is the ending vertex of the tree.
d)	The derangement is a rearrangement or permutation such that no number appears in its original position.

e) The rook polynomial for the board □ is 1+x.

SECTION B - K3 (CO2)

Answer any TWO of the following in 100 words **(2 x 10 = 20)**

5. Suppose that $t(n, n-1) = 1$ and $(n-k-1)\,t(n,k) = k(n-1)\,t(n,k+1)$ for each $k < n-1$. Show that $t(n,k) = \frac{(n-1)^{n-k-1}(n-2)!}{(k-1)!(n-k-1)!}$.

6. Show that $\binom{8}{3} = \binom{8}{5}$ *and* $\binom{n}{n-2} = \binom{n}{2} = \frac{1}{2}n(n-1)$ ·

7. Use generating function, to find the recurrence relation of $a_n = 4a_{n-1} + 4a_{n-2} - 164a_{n-3}$ with the given boundary conditions $a_1 = 8, a_2 = 4, a_3 = 24$.

8. Derive $a_n = n!\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \cdots + (-1)^n \frac{1}{n!}\right\}$ by using inclusion and exclusion principle.

SECTION C – K4 (CO3)

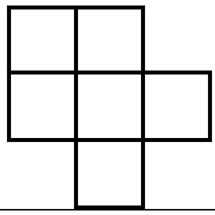
Answer any TWO of the following in 100 words **(2 x 10 = 20)**

9. Show that $\binom{n}{r} = \binom{n}{n-r}$, $0 \le r \le n$.

10. Let S be a set of mn objects. Prove that 'S' can be split up into n sets of m elements in $\frac{(mn)!}{(m!)^n n!}$ different ways.

11. Derive the general formula for U_n, i.e., the number of different rooted trees.

12. Find the rook polynomial of the board



SECTION D – K5 (CO4)

Answer any ONE of the following in 250 words **(1 x 20 = 20)**

13. (a) Suppose that each of $k-$ indistinguishable golf balls have to be coloured with any one of n colours using binomial theorem (generating function) approach. Find out how many different colouring are possible and hence deduce the case $k = 4$ and $n = 9$.

(b) Solve the following Assignment Problem to find the optimal assignment schedule:

	A	B	C	D
I	5	7	15	12
II	8	3	9	10
II	4	14	2	5
IV	6	3	1	14

14. (a) State and prove Marriage Theorem.

(b) Let n be a positive integer. Show that if $(1+x)^n$ is expended as a sum of powers on n, the coefficient of x^r is $\binom{n}{r}$.

SECTION E – K6 (CO5)

Answer any ONE of the following in 250 words **(1 x 20 = 20)**

15. (a) If a football league of n teams, each team plays each other twice. The number of games played is therefore 2C. where C is the number of ways choosing two objects from n given objects. Prove

	that $C = (n-1) + (n-2) + \cdots \ldots + 2 + 1 = \frac{n(n-1)}{2}$ and deduce the number of games played in a league of 22 teams. (b) Explain ordered selection and evaluate the following: (i) $p(7,4)$, (ii) $p(9,5)$
16.	(a) Suppose that a_1 and a_2 are given, then $a_n = Aa_{n-1} + Ba_{n-2}, (n \geq 3)$, holds. Then prove the following: (i) if the roots α, β of the equation $x^2 = Ax + B$are distinct, then $a_n = k_1\alpha^n + k_2\beta^n$, where the constants k_1, k_2 are determined uniquely by a_1 and a_2. (ii) if $x^2 = Ax + B$has repeated root α, then $a_n = (k_1 + nk_2)\alpha^n$ (b) Given a chessboard C, choose any square of C and let D denote the board obtained by deleting from C every square in the same row or column at the chosen square (including the chosen square itself). Let E denote the board obtained from C by deleting only the chosen square. Then prove that $R(x, C) = xR(x, D) + R(x, E)$.

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